Differenciálegyenletek 1. zh. megoldások

(1) $y' = \dfrac{x}{x^2-1} \cdot \dfrac{y^2-1}{y}$ $\quad y(2)=1 \Rightarrow y \equiv 1$ $\quad$ 2p

$\int \dfrac{y}{y^2-1}\,dy = \int \dfrac{x}{x^2-1}\,dx \Rightarrow$ $\quad y(2)=2 \Rightarrow y = x$ $\quad$ 2p

(2) $y' - \frac{1}{x}y = x^2 e^{x^2}$ $\quad y_h(x) = cx$ 1p $\quad y_p(x) = c(x)x$

$\Rightarrow c'(x) = xe^{x^2}$ $\quad c(x) = \frac{1}{2}e^{x^2} \Rightarrow y_p(x) = \frac{1}{2}xe^{x^2}$ $\quad$ 2p

$y_{ált}(x) = cx + \frac{1}{2}xe^{x^2}$ $\quad y(1)=1 \Rightarrow$ $\quad y(x) = \left(1 - \frac{e}{2}\right)x + \frac{1}{2}xe^{x^2}$ $\quad$ 1p

(3) $y_h(x) = e^{\lambda x}$ $\quad \lambda^4 - 1 = 0$ $\quad (\lambda-1)(\lambda+1)(\lambda^2+1) = 0$

$\lambda_1 = 1 \quad \lambda_2 = -1, \quad \lambda_{34} = \pm i$

$\Rightarrow y_h(x) = C_1 e^x + C_2 e^{-x} + C_3 \sin x + C_4 \cos x$ $\quad$ 2p

$\{e^x\} \quad \{x^2, x, 1\} \rightarrow \{xe^x\} \quad \{x^2, x, 1\}$

$y_p(x) = Axe^x + Bx^2 + Cx + D \Rightarrow y_p(x) = \frac{1}{4}xe^x + x^2$

$\Rightarrow y_{ált}(x) = y_h(x) + y_p(x)$ $\quad$ 2p

(4) $f(\lambda, t) = e^{\lambda t} = h(\lambda, t) = h_2(t)\lambda^2 + h_1(t)\lambda + h_0(t)$

$\lambda_1 = 0 \Rightarrow h_0(t) = 1$; $\lambda_2 = 0 \Rightarrow h_1(t) = t$; $\lambda_3 = 1 \Rightarrow h_2(t) = e^t - t - 1$ $\quad$ 2p

$e^{\underline{\underline{A}}t} = (e^t - t - 1)\underline{\underline{A}}^2 + t\,\underline{\underline{A}} + \underline{\underline{E}} = \begin{pmatrix} 1 & t & 2e^t - t + 2 \\ 0 & 1 & e^t - 1 \\ 0 & 0 & e^t \end{pmatrix}$ $\quad$ 2p

$\underline{x}(t) = e^{\underline{\underline{A}}t} \cdot \underline{c}$

(5) Egzakt $\quad (y^3 + 2xy + y)\,dx + (3xy^2 + x^2 + x)\,dy = 0$

$F(x,y) = xy^3 + x^2y + xy$ $\quad$ 2p

mo $\quad xy^3 + x^2y + xy = C$; $y(1) = 1 \Rightarrow xy^3 + x^2y + xy = 3$

2p