Mat G3 2. zh. megoldások

① a) $\int \frac{y}{y^2-9}\,dy = \int \frac{x}{x^2+1}\,dx \Rightarrow \frac{1}{2}\ln|y^2-9| = \frac{1}{2}\ln(x^2+1) + C$ 2p

4p $|y^2-9| = C(x^2+1) \Rightarrow C = 4$ 2p

$\Rightarrow y^2(x) = 5 - 4x^2$

b) $y \equiv 3$ 2p

② $y' - \frac{1}{x}y = x^2e^x$ $y_h(x) = Cx$ 2p $y_p(x) = C(x)\cdot x$

$\Rightarrow C(x) = xe^x - e^x$ 4p $y_{ält}(x) = Cx + x^2e^x - xe^x$ $C = 1$ 2p

$\Rightarrow y(x) = x + x^2e^x - xe^x$ 2p

③ Egzakt. 2p $F(x,y) = 3x^2y^2 + 3xy + x^2 + 2y^2$ 6p

mo $F(x,y) = C \Rightarrow C = 9. \Rightarrow 3x^2y^2 + 3xy + x^2 + 2y^2 = 9$ 2p

④ $y_h(x) = C_1\cos x + C_2\sin x$ 2p

4p $y_p(x) = Ax\cos x + Bx\sin x \Rightarrow B = 1,\ A = 0$

$\Rightarrow y_p(x) = x\sin x$ $. \Rightarrow y_{ält}(x) = C_1\cos x + C_2\sin x + x\sin x.$ 4p

⑤

$\lambda_1 = 4$ $\lambda_2 = -2$ 4p

$\underline{s}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\underline{s}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ 4p

$\Rightarrow \underline{x}(t) = C_1e^{4t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2e^{-2t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ 2p